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Dynamic Snow Plow Fleet Management under Uncertain Demand and Service Disruption

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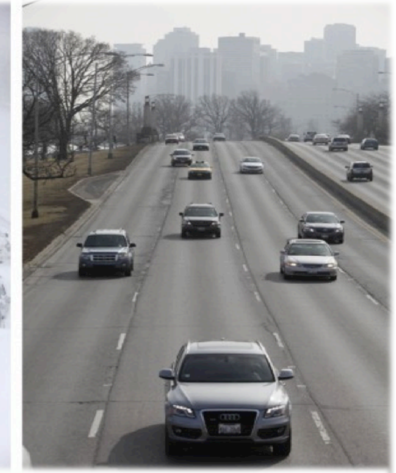
Motivation

- Safety impact of snow and ice storms (USDOT, 2014)
  - 24% of annual weather-related vehicle crashes on snowy or icy pavement
  - Over 1,300 fatal crashes in vehicle crashes on snowy or icy roads
- Economic implications
  - \$300-\$700 M for 1-day shutdown (American Highway Users Alliance, 2010)
  - 20% of state DOT maintenance budgets for snow and ice control (USDOT, 2014)
  - Over \$110,000 cost per shift on average for snow removal operations (District of Columbia, 2010)



Contributions

Strategic Infrastructure Decisions

- Resource planning and allocation:
  - Resource replenishment facilities
  - Roadway capacity expansion

Snow Plow Route Planning

- Route planning
- Fleet assignment
- Resource replenishment

Dynamic Fleet Management

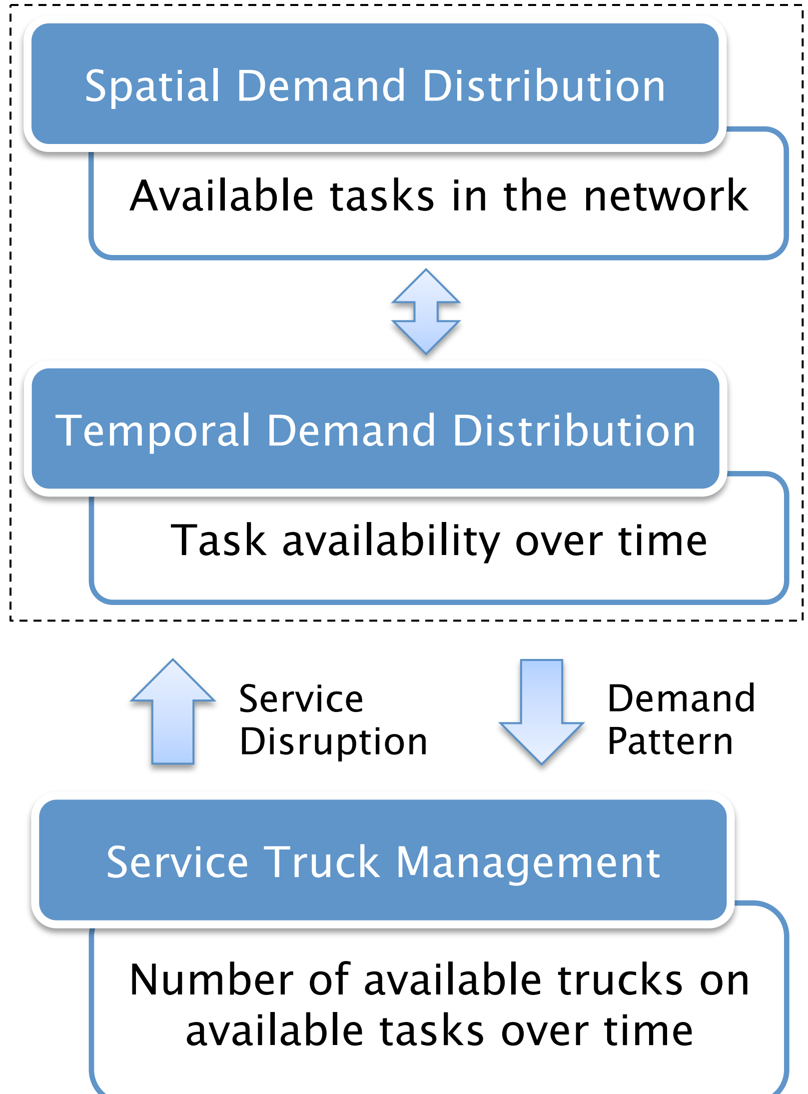
- Dynamic fleet scheduling in long-storm conditions:
  - Uncertain demand
  - Service disruption

- Decision support tool

Outline

- Problem Statement
- Background
- Model Development
- Solution Approach
- Numerical Results
- Summary

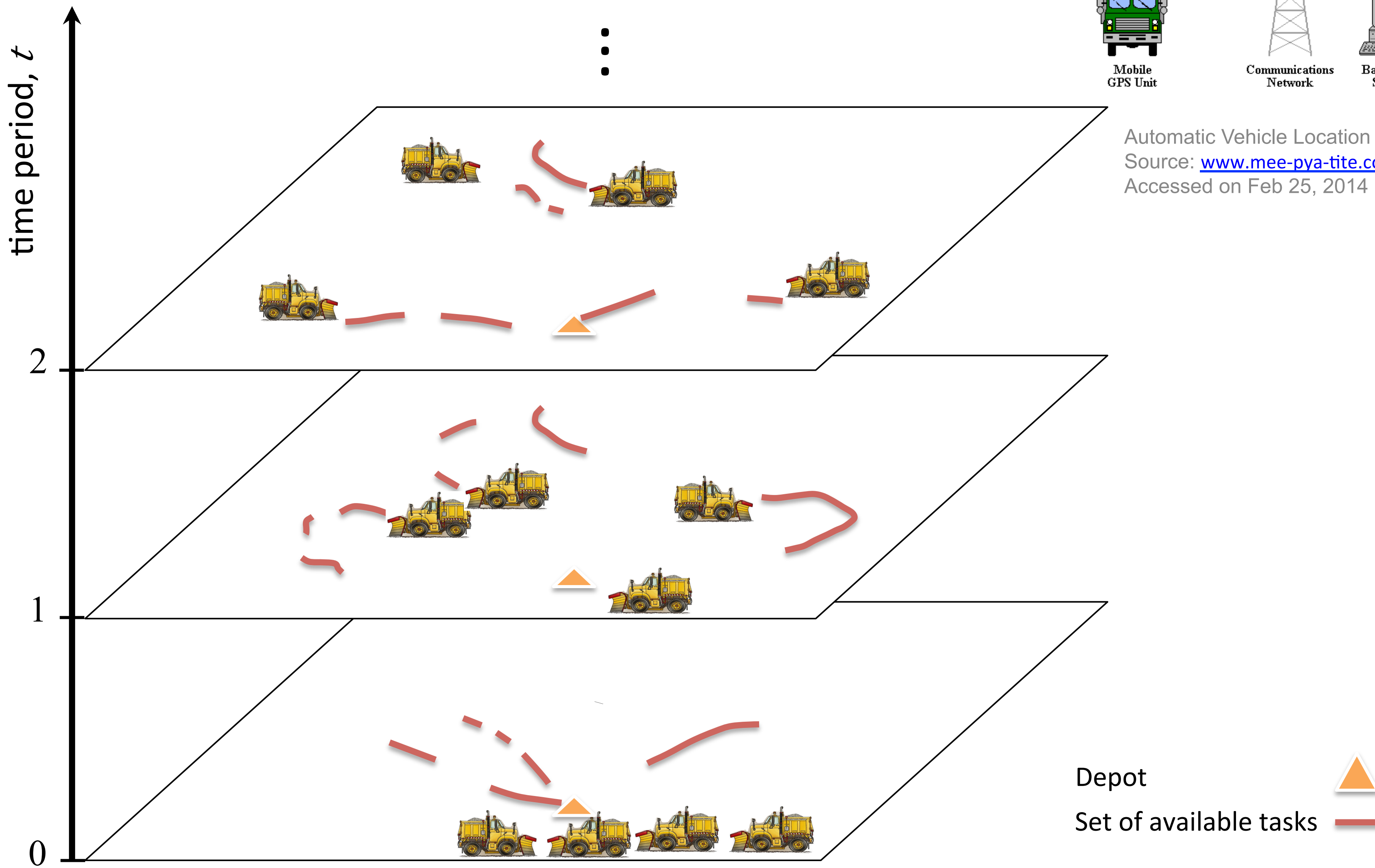
Problem Statement

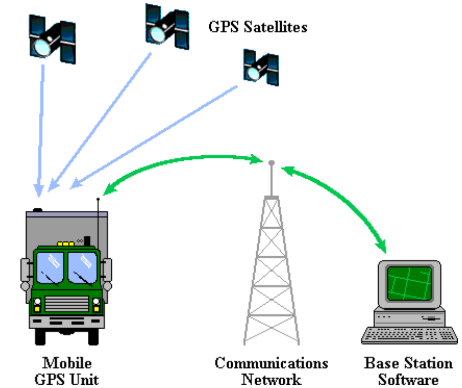


Problem Statement

- Considerations:
  - Random availability of the tasks and trucks
    - New trucks and tasks become available via some random process
  - Truck repositioning
    - There is no available task
    - Tasks at other route have higher priority while service is disrupted
  - Truck deadheading

Problem Statement





Automatic Vehicle Location (AVL)
Source: www.mee-pya-tite.com,
Accessed on Feb 25, 2014

Background

- Dynamic Programming (DP) Methods

Method	Objective/Focus	Researcher
Traditional DP	Discrete state and action spaces	Puterman 1994
Forward DP	Monte Carlo	Bertsekas and Tsitsiklis 1996, Sutton and Barto 1998

- Not as sensitive to large state spaces, but suffer from large action spaces
- Require the ability to estimate the value of the system being in a particular state
- Convergence proofs are only available under very strong assumptions

Background

- Stochastic Linear Programming Methods (Infanger 1994, Kall & Wallace 1994, Birge & Louveaux 1997, Powell 2002)

Method	Objective/Focus	Researcher
Two-stage stochastic linear programs	Large-scale optimization problems subject to non-anticipativity constraints	Dantzig 1955, Rockafellar & Wets 1991
	Approximate the second-stage recourse function (linearization methods, static/dynamic sampling)	Ermoliev 1988, Ruszczynski 1980, Van Slyke & Wets 1969, Higle & Sen 1991
Multistage problems	Nested Benders algorithm	Birge 1985
	Sampling-based methods	Higle & Sen 1991, Pereira & Pinto 1991, Chen & Powell 1999

Background

- Approximation Methods in the Context of Fleet Management

Method	Objective/Focus	Researcher
Methods for continuous flows	Designed for non-integer flows	Jordan & Turnquist 1983, Powell 1986
Approximations that naturally produce integer solutions	Not designed for problems with time windows (difficult to apply)	Powell 1987, Frantzeskakis & Powell 1990, Cheung & Powell 1996, Powell & Carvalho 1998
Linear approximation with a multiplier adjustment procedure (LAMA)	Works only on deterministic problems	Carvalho & Powell 2000
CAVE (Concave Adaptive Value Estimation) algorithm	• Construct a concave, separable, piecewise-linear approximation of value function • More flexible and responsive than linear approximations	Godfrey & Powell 2001, 2002

Background

- Online Vehicle Routing Problem

Method	Objective/Focus	Researcher
Online routing algorithms w/o service flexibility and rejection options	Minimize the time to visit a set of locations that are revealed incrementally over time	Jaillet & Wagner 2007, Jaillet & Lu 2011, Yang
Rolling horizon technique using a mixed integer programming formulation for the offline version	Online fleet assignment and scheduling Minimize costs of empty travels, jobs' delayed completion times, and job rejections w/o time windows	Yang, Jaillet, & Mahmassani 1998, 2002, 2004
Simulation framework using real-time info about vehicle locations and demands	Dynamic dispatching, load acceptance, and pricing strategies	Regan, Mahmassani, & Jaillet 1996

Model Development

- Network:

  T: the number of time periods in the planning horizon

  $\Gamma = \{0, 1, \ldots, T-1\}$, the times at which decisions are made

  Ψ: the set of physical routes in the network

Model Development

- Trucks and Tasks:

For each $t \in \Gamma$ and $i \in \Psi$,

$\hat{\kappa}_{it}$: the number of trucks that first become available on route i at time t; $\hat{\kappa}_t = (\hat{\kappa}_{it})_{i \in \Psi}$

κ_{it} : the number of trucks available on route i at time t before any new arrivals have been added in; $\kappa_t = (\kappa_{it})_{i \in \Psi}$

κ_{it}^{+}: the total number of trucks that are available to be used on route i at time t

$\hat{A}_{it}$: the set of tasks that first become available on route i at time t; $\hat{A}_t = (\hat{A}_{it})_{i \in \Psi}$

A_{it} : the set of tasks available on route i at time t before the new arrivals in $\hat{A}_{it}$ are added to the system; $A_t = (A_{it})_{i \in \Psi}$

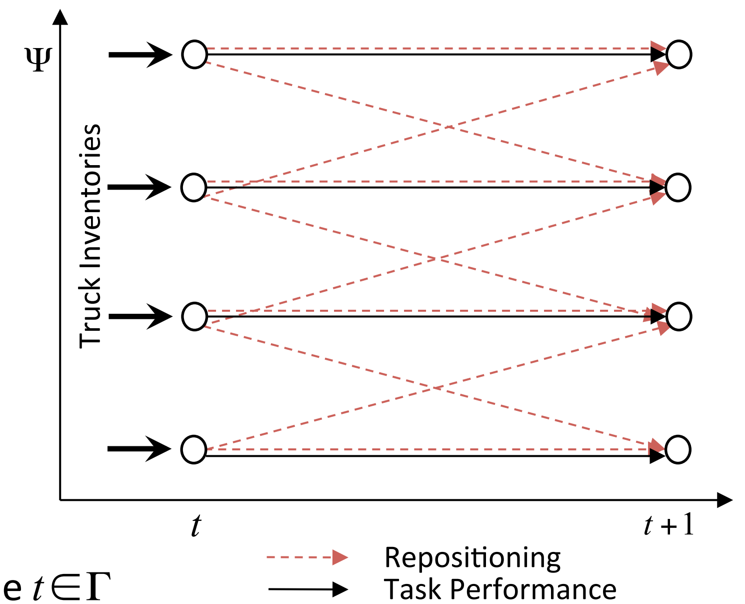
A_{it}^{+}: the set of tasks available to be services on route i at time t, including the newly popped-up tasks

A_{it}^{d}: the set of deadheads required to reach the available tasks on route i at time t

$\mathrm{W}_t = (\hat{\kappa}_t, \hat{A}_t)$, represents the new info arriving in time period t

$(\mathrm{W}_t)_{t=0}^{T}$: stochastic info process, with realization $\mathrm{W}_t(\omega) = \omega_t = (\hat{\kappa}_t(\omega), \hat{A}_t(\omega))$

Model Development



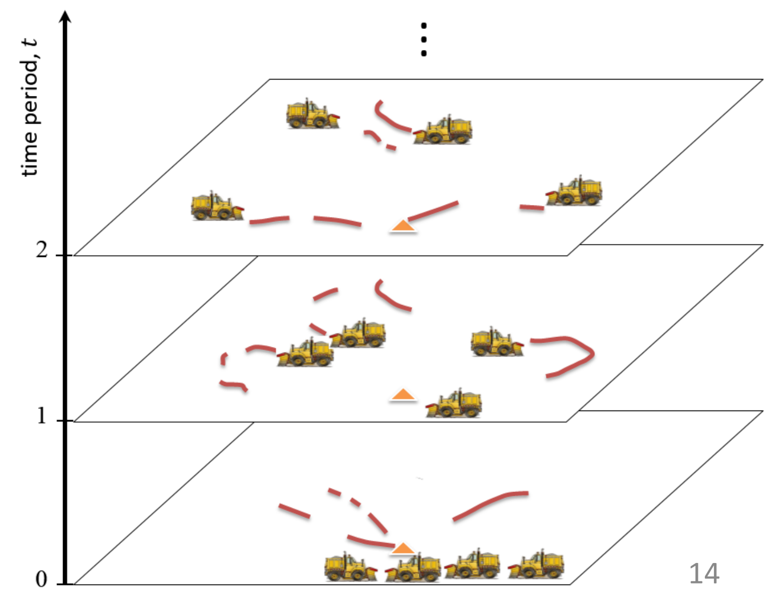
- Decision Variables:

$$\mu_i^t = \begin{cases} 1, & \text{if a truck travels on route } i \in \Psi \text{ at time } t \in \Gamma, \\ 0, & \text{o. w.} \end{cases}$$

η_{ij}^t = number of trucks reposition from $i \in \Psi$ to $j \in \Psi$ at time $t \in \Gamma$

- State Variables:

$$S_t = \{\kappa_t, A_t\}$$



Model Development

- Objective Function:

  c_{ij}^{p} : cost of repositioning from route i to route j $(\$ / mile)$

  c_{a}^{r} : the benefit from plowing task $a \in A_{it}^{+}$ $(\$ / mile)$

  c_{a}^{d} : the cost of deadheading link $a \in A_{it}^{d}$ $(\$ / mile)$

  $\lambda_{i,r}^{t}$: the number of tasks on route i at time t

  $\lambda_{i,d}^{t}$: the number of deadheads required to reach the tasks on route i at time t

$$f_t(\mu_t, \eta_t) = \overbrace{\boxed{\sum_{i \in \Psi} \left\{ \mu_i^t \left(\sum_{a \in \mathcal{A}_{it}^{+}} c_a^r \lambda_{i,r}^t - \sum_{a \in \mathcal{A}_{it}^{d}} c_a^d \lambda_{i,d}^t \right)\right.}}^{\text{Total benefit gained from decisions at time } t} - \overbrace{\boxed{\left.\sum_{i \in \Psi} \sum_{j \in \Psi} c_{ij}^{p} \eta_{ij}^{t} \right\}}}^{\text{Total repositioning cost at time } t}, \qquad (1)$$

subject to

$$\sum_{j \in \Psi} \eta_{ij}^t(\omega) + \sum_{i \in \Psi} \mu_i^t(\omega) = \mathcal{K}_{it} + \widehat{\mathcal{K}}_{it}, \ \forall i \in \Psi, \qquad (2)$$

$$\mu_i^t(\omega) \in \{0, 1\}, \forall i \in \Psi, \text{ and} \qquad (3)$$

$$\eta_{ij}^t(\omega) \geq 0, \forall i, j \in \Psi\,, \qquad (4)$$

Solution Approach

A_t^e : the set of tasks that expire in time period t

System Dynamics

$$\mathcal{A}_{t+1} = \mathcal{A}_t^+ \setminus \mathcal{A}_t^e, \quad (5)$$

$$\mathcal{K}_j^{t+1}(\omega) = \sum_{i \in \Psi} \eta_{ij}^t(\omega) + \sum_{j \in \Psi} \mu_j^t(\omega), \ \forall j \in \Psi, \quad (6)$$

Dynamic Programming

$$\underset{\mu_0, \eta_0}{\text{maximize}} \ f_0(\mu_0, \eta_0) + \mathbb{E}\left\{ \sum_{t \in \Gamma \setminus 0} \underset{(\mu_t, \eta_t)}{\text{maximize}} \ f_t(\mu_t, \eta_t) \right\}$$

$$\Downarrow$$

Bellman Eq.

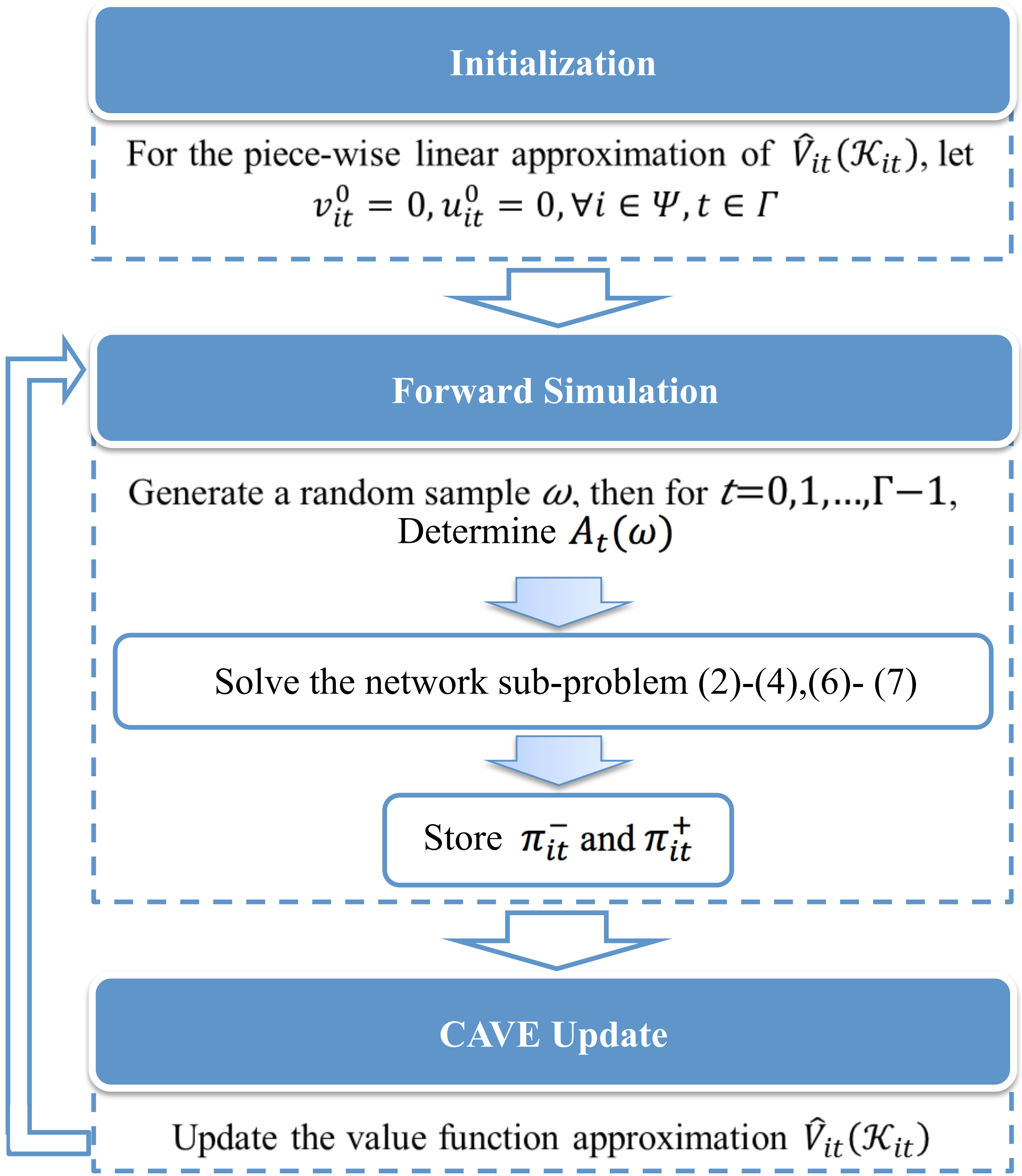
$$\boxed{\widetilde{V}_t(\mathcal{K}_t, \omega) = \underset{(\mu_t(\omega), \eta_t(\omega))}{\text{maximize}} \ f_t(\mu_t(\omega), \eta_t(\omega)) + \boxed{\widehat{V}_{t+1}(\mathcal{K}_{t+1}(\omega)),}} \quad (7)$$

subject to

(2)-(4) and (6)

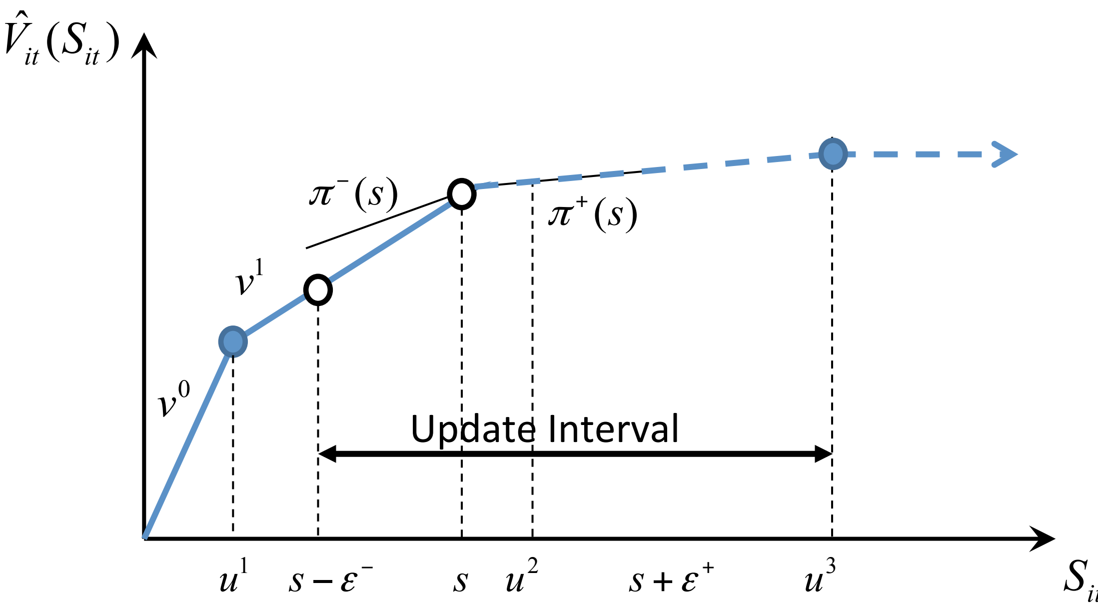
$$\hat{V}_t(\kappa_t) = \sum_{i \in \Psi} \hat{V}_{it}(\kappa_{it})$$

Value Function Approximation


Initialization
For the piece-wise linear approximation of $\hat{V}_{it}(\mathcal{K}_{it})$, let
$v_{it}^{0} = 0, u_{it}^{0} = 0, \forall i \in \Psi, t \in \Gamma$
Forward Simulation
Generate a random sample ω, then for t=0,1,...,Γ−1,
Determine $A_t(\omega)$
Solve the network sub-problem (2)-(4),(6)- (7)
Store π_{it}^{-} and π_{it}^{+}
CAVE Update
Update the value function approximation $\hat{V}_{it}(\mathcal{K}_{it})$

Solution Approach

- Fitting Concave Functional Approximations (CAVE*)



$v_{it}^{\ 0} = 0, u_{it}^{\ 0} = 0$

Initialize parameters $\varepsilon^-, \varepsilon^+, \alpha$

Find gradients π_{it}^- and π_{it}^+ for a given ω

$$n_{it}^{\ -} = \min\left\{n \in N : v_{it}^{\ n} \le (1-\alpha)v_{it}^{\ n+1} + \alpha\pi_{it}^{\ -}\right\}$$

$$n_{it}^{\ +} = \min\left\{n \in N : v_{it}^{\ n} < (1-\alpha)v_{it}^{\ n-1} + \alpha\pi_{it}^{\ +}\right\}$$

$$UI = \left[\min\left\{\kappa_{it} - \varepsilon^-, u^{n_{it}^-}\right\}, \max\left\{\kappa_{it} + \varepsilon^+, u^{n_{it}^+}\right\}\right)$$

Create new break points

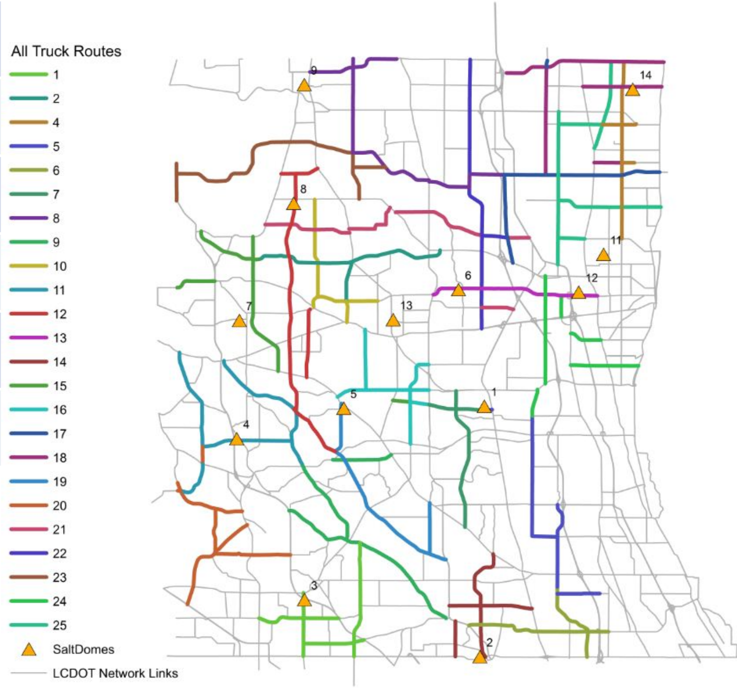
$$v_{it,new}^{n} = \alpha\pi_{it} + (1-\alpha)v_{it,old}^{n},$$

where $\begin{cases} \pi_{it} = v_{it}^{\ -}, \text{ if } u_{it}^{\ n} < \kappa_{it} \\ \pi_{it} = v_{it}^{\ +}, \text{ o.w.} \end{cases}$

* Concave Adaptive Value Estimation

Numerical Results

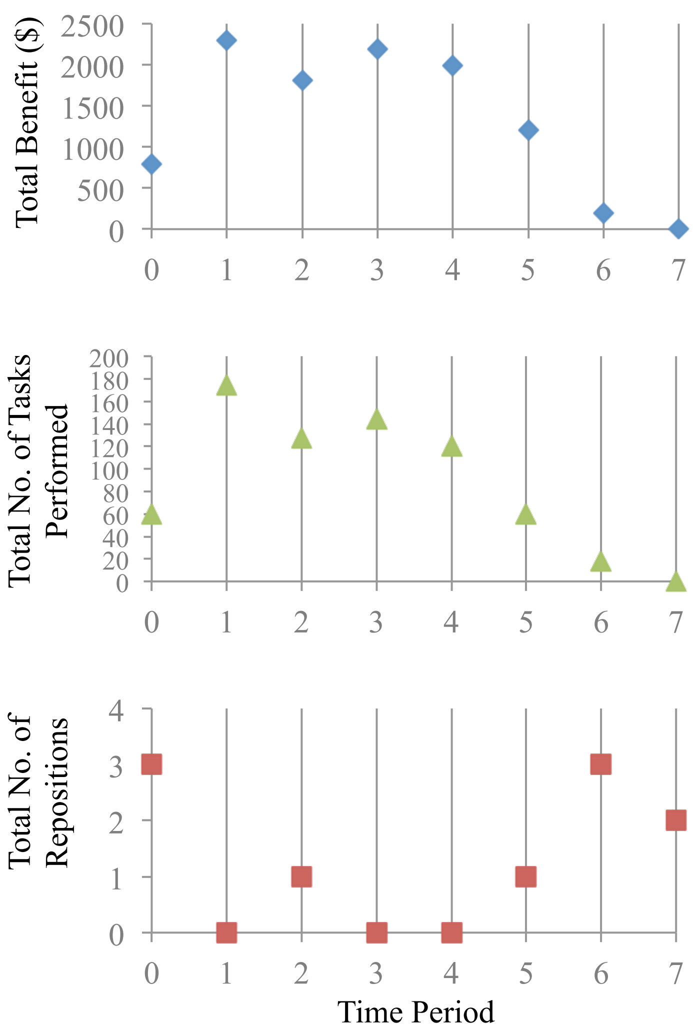
Problem Characteristics	Attribute Values
number of routes	Lake County, IL optimal truck routes
number of trucks	equal to the number of routes, but subject to failure
planning horizon length, T	8 time periods
number of tasks over simulation	from Lake County, IL task links*
time period length (fixed)	20 min
net task revenue per mile	$10
repositioning cost per mile	$1
deadhead cost per mile	$1



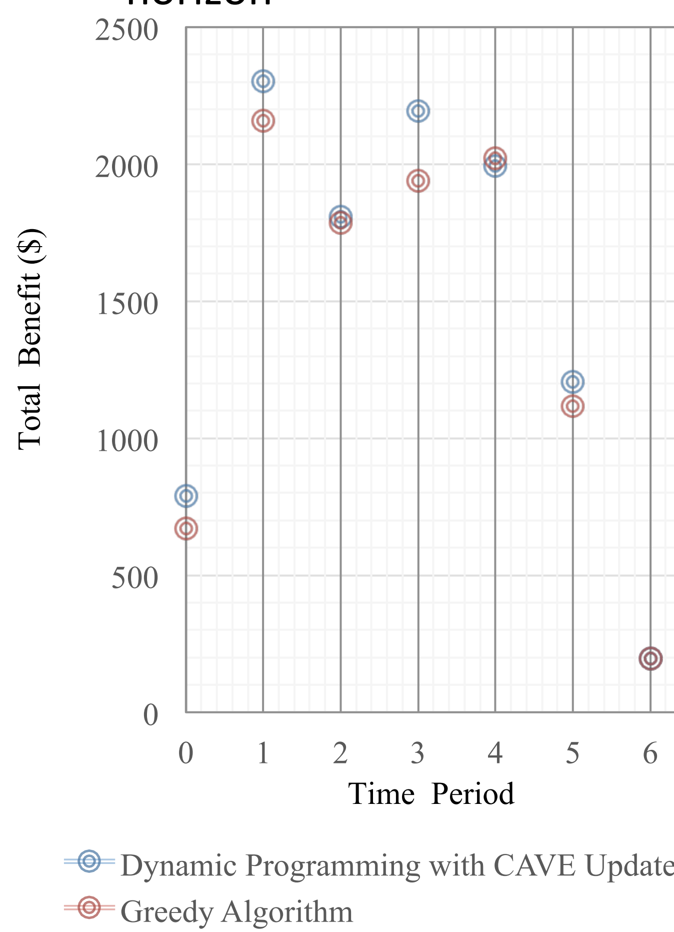
- Tasks become available over time on routes

- The algorithm is coded in C++ and run on a desktop computer with 2.67 GHz CPU and 3 GB memory
- CPLEX is called to solve the forward simulation.
- Number of routes in the last iteration = 14

Numerical Results



- Comparison with alternative algorithms
  - 5.8% difference over the planning horizon



Summary

- Dynamic fleet management for snow control activities under uncertainty (operations)
  - Approximate Dynamic Programming (ADP) including a forward simulation followed by an update
  - Case study based on LCDOT truck routes
  - Comparison with a greedy approach

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Thanks

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